## Annex F

# Derivation of revised estimate for subarea 7 in 1991 and zero abundance estimates 

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An estimate of abundance sub-area $7 \mathrm{~W}^{3}$ in 1991 used in the 2003 trials was actually an estimate developed from the combination of results of surveys in 1990, 1991 and 1992. It is not acceptable to derive estimates for the component subareas ( $7 \mathrm{CN}, 7 \mathrm{CS}$ and 7 WR ) by splitting the estimate proportional to sub-area size because the sighting rates in the three sub-areas had been very different. These data were reanalysed in a manner that took account of this difference and the resultant alternative for splitting the overall abundance estimate between the three sub-areas was agreed for use in projections for the ISTs.

Table 1 shows the abundance prorated by $n A / L$ from total estimate. The two estimates for each subarea were averaged to give the following estimates for use in trials: $7 \mathrm{CS} 0 ; 7 \mathrm{CN}$ $853 \mathrm{CV}=0.23$; 7WR $311 \mathrm{CV}=0.23$.

## Inclusion of zero abundance estimates in the trials

Table 1 includes one abundance estimate which is zero. Annotation (29) of the RMP specification document (IWC, 2012) specifies how a Poisson likelihood component is developed in cases when a zero abundance estimate occurs. The annotation says:
(29) An example where the lognormal assumption cannot be used is when the estimate of absolute abundance is zero. Zero estimates of absolute abundance arise when no sightings of the target species are made on primary effort during a survey of an area. This should not be a frequent occurrence, but such estimates should not be ignored when they do occur.
${ }^{3}$ Subarea 7W was used in the 2003 trials and is a combination of the current sub-areas $7 \mathrm{CS}, 7 \mathrm{CN}$ and 7 WR .

Although several factors contribute to the variance of an estimate of absolute abundance, the variance is dominated by the variance in the number seen when the number of sightings is very low. The variance of the number of sightings will be at least as high as the variance of a random variable with a Poisson distribution with expectation equal to the expectation of the number of sightings. The number of sightings refers to the number of schools or groups, rather than to individual animals.
The expected number of sightings, $E(n)$, is proportional to the true absolute abundance, $P: E(n)=P / \alpha$
The parameter $\alpha$ represents the estimate of absolute abundance that would have been obtained had there been exactly one sighting. This will be a function of the survey effort, the size of the area, and survey parameters that may need to be estimated by adopting values from similar surveys. Ignoring the variance of $\alpha$, the likelihood of the zero estimate of absolute abundance is the following function of the true absolute abundance:
$L(P)=\exp (-P / \alpha)$
Since the only covariance between the absolute abundance estimate and other absolute abundance estimates is that due to the $\alpha$ parameter, whose variance is being ignored, the joint likelihood function of the zero estimate of absolute abundance and the remaining estimates is taken to be the product of the respective likelihood functions.
The information about the zero estimate of absolute abundance that needs to be supplied to the Catch Limit Algorithm is: (i) the year of the zero estimate; (ii) the fact that it is a zero estimate; and (iii) the value of the $\alpha$ parameter. The computer program implementing the Catch Limit Algorithm that has been validated by the IWC Secretariat has the facility to handle zero estimates of absolute abundance in this manner. $P$ is identified with the simulated population size generated by the Catch Limit Algorithm's internal calculations.
Since the treatment above ignores some contributions to the variance of a zero estimate of absolute abundance, it assigns more weight to a zero estimate than is strictly warranted.


Fig. 1. Track line on effort (black thick line), primary sighting (triangle), sub-area definition (dotted thick line) and area definition for estimate (grey thick line) for Shunyo Maru in 1991 (left) and 1992 (right).

Table 1
Abundance prorated by $n A / L$ from total estimate ( 1,164 animals, $\mathrm{CV}=0.183$ ).

|  | 91 Shunyo Maru |  |  | 92 Shunyo Maru |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 7 CN | 7CS | 7WR | 7 CN | 7CS | 7WR |
| $L$ : Research distance (n.miles) | 775 | 516 | 597 | 703 | 774 | 816 |
| $n$ : no. of primary sightings | 11 | 0 | 1 | 6 | 0 | 2 |
| $A$ : Area (n.miles ${ }^{2}$ ) | 15,948 | 26,828 | 26,088 | 16,545 | 26,826 | 34,232 |
| $n / L * A$ | 226.3483 | 0 | 43.67138 | 141.2217 | 0 | 83.89933 |
| $P$ | 976 | 0 | 188 | 730 | 0 | 434 |
| Coverage (\%) | 87.2 | 100 | 4.03 | 90.5 | 100 | 29.2 |

Table 2
Population estimates to replace zero estimates in the trials.

| Sub-area | 6 E |  |  | 10E |  |  | 10W | 7 CN |  | 7WR |  | 7E | 8 |  | 11 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Season | 2002 | 2003 | 2004 | 2002 | 2003 | 2005 | 2006 | 1991 | 1992 | 1991 | 1992 | 2006 | 2006 | 2007 | 2003 | 2007 |
| $L$ | 1,676 | 1,226 | 1,037 | 486 | 651 | 466 | 1,157 |  |  |  |  | 461 | 1,039 | 914 | 192 | 564 |
| $n$ | 21 | 19 | 7 | 10 | 7 | 9 | 36 | 11 | 6 | 1 | 2 | 2 | 3 | 2 | 10 | 19 |
| A | 71,914 | 71,914 | 71,914 | 27,823 | 27,823 | 17,912 | 63,912 |  |  |  |  | 48,208 | 162,789 | 162,789 | 15,243 | 9,064 |
| $P$ | 891 | 935 | 727 | 816 | 405 | 599 | 2,477 | 976 | 730 | 188 | 434 | 247 | 309 | 391 | 882 | 377 |
| Scaled | 18.1 | 21.0 | 44.2 | 34.8 | 24.6 | 28.4 | 29.3 | 37.8 | 51.8 | 80.1 | 92.4 | 52.6 | 43.9 | 83.3 | 37.6 | 8.5 |
| Average | 27.8 |  |  | 29.3 |  |  | 29.3 | 44.8 |  | 86.3 |  | 52.6 | 63.6 |  | 23.0 |  |

For the zero abundance estimate obtained above for subarea 7CS in 1991, there is a final output of a negative $\log$ - likelihood component of $P / 98.6$ where $P$ is the true abundance present. This could not, however, be used directly in the $I S T$ s as the program implementing the RMP (which is also used for the $I S T$ s) does not make allowance for such terms. Accordingly the Workshop agreed to replace this form with a negative log-likelihood based on the assumption of a log-normally distributed pseudo estimate, which as with the Poisson form would yield a value of 1when $P=98.6$. Since this is not sufficient to define this likelihood term unambiguously, the Workshop decided to fix the mean at 42 (Adams, 1995) which resulted in a standard deviation of 0.603 . This approach is also to be applied to other cases of zero abundance estimates which may occur in the projections as well.

These other sub-areas with zero abundances, either in the past or in future projections are to be accorded negative loglikelihoods with the same standard deviation, but a different mean depending on the what the population estimates would have been for recent surveys in those areas had there been only one minke whale sighting made. Specifically, with averages taken over such population estimates calculated separately for each of the surveys listed and then scaled by 42/98.6, the results are given in Table 2.

## REFERENCES

Adams, D. 1995. Hitchhikers Guide to the Galaxy: a Trilogy in Five Parts. William Heinemann, London. 784pp.
International Whaling Commission. 2012. Requirements and Guidelines for Implementations under the Revised Management Procedure. J. Cetacean Res. Manage. (Suppl.) 13:495-506.

